ENERGY DISSIPATION RATE IN A BAFFLED VESSEL WITH PITCHED BLADE TURBINE IMPELLER*

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Mechanical energy dissipation was investigated in a cylindrical, flat bottomed vessel with four radial baffles and the pitched blade turbine impeller of varied size. This study was based upon the experimental data on the hydrodynamics of the turbulent flow of water in an agitated vessel. They were gained by means of the three-holes Pitot tube technique for three impeller-to-vessel diameter ratio $d/D = 1/3$, $1/4$ and $1/5$. The experimental results obtained for two levels below and two levels above the impeller were used in the present study. Radial profiles of the mean velocity components, static and total pressures were presented for one of the levels. Local contribution to the axial transport of the agitated charge and energy was presented. Using the assumption of the axial symmetry of the flow field the volumetric flow rates were determined for the four horizontal cross-sections. Regions of positive and negative values of the total pressure of the liquid were indicated. Energy dissipation rates in various regions of the agitated vessel were estimated in the range from 0·2 to 6·0 of the average value for the whole vessel. Hydraulic impeller efficiency amounting to about 68% was obtained. The mechanical energy transferred by the impellers is dissipated in the following ways: 54% in the space below the impeller, 32% in the impeller region, 14% in the remaining part of the agitated liquid.

An impeller rotating in a vessel filled with a liquid causes the mechanical energy to be supplied into the agitated liquid. The energy input forces the liquid to circulate. That circulating flow promotes in consequence heat and mass transfer processes in an agitated vessel. Axially pumping impellers are generally efficient in those applications where intensive circulation should be generated at a relative low power consumption. The knowledge of the global and local energy dissipation rate can be important in the quantitative description of the rate of turbulent transfer of momentum, heat and mass in agitated vessels. Experimental data on the local dissipation are rather rare and only two papers were found in the literature: by Fořt et al.[1] and by Ranade et. al.[2].

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The authors[1] have studied the problem on the basis of the Pitot tube measurements made for one level below and one level above the impeller in the vessel of the diameter $D = 0{\cdot}29$ m. The impellers of the diameter d equal to $D/3$, $D/4$ or $D/5$ were located with their center at the distance $H_2 = D/4$ above the vessel bottom. The hydraulic efficiency of the impellers has been found in the range 60 to 70%. About 90% of the energy transferred to the agitated liquid has been dissipated in the region below the impeller and 10% of the energy in the rest of the agitated charge.

Rande et al.[2] have reported a computed profile of the turbulent energy dissipation rate ε obtained from the numerical simulations of the turbulent flow in a baffled vessel with the pitched blade turbine impeller. They predicted that the local values of ε steeply decreased in the impeller discharge stream flowing in the space between the impeller and the bottom. In the remaining vessel volume the ε values have varied less markedly. The authors[2] have suggested that 30% of the energy supplied to the impeller has got dissipated in the impeller region, 57% in the space below the impeller and 13% above it. Their experimental technique, however, has not allowed them to verify the distribution computed.

The objective of that work was to gain more detailed information on the distribution of the energy dissipation rate within the vessel agitated by the pitched blade turbine impellers. Integral measures of the volumetric and energy flow rates should be computed for the upward and downward flow directions. Then, radial profiles of the mean velocity components and pressures can be obtained for a few subregions of the system mentioned. Local contribution of the mass and energy fluxes in the agitated vessel will be also investigated.

THEORETICAL

The standard geometry of the experimental system was considered (see Fig. 1). The system consisted of a cylindrical vessel with flat bottom, four radial baffles and a pitched blade turbine impeller located axially. The impeller had the varied diameter d and pumped liquid downward. A system of coordinates, radial r and axial z, was proposed with its origin in the bottom centre. The turbulent flow of the agitated liquid was studied in consideration of the following assumptions:
– agitated charge is a homogeneous Newtonian liquid with a constant density ϱ;
– turbulent flow is fully developed, i.e. quasistationary;
– the investigated fields of velocity and pressure can be regarded as axisymmetrical;
– only the following energy forms: internal (expressed by static pressure), kinetic and potential should be considered by means of the mechanical energy balance of the moving liquid.

A sign of the radial w_r and axial w_z components of the mean velocity vector were chosen in accordance with direction of the respective coordinate axis shown in Fig. 1.

Four sets of measurements were considered, each of them referred to different level z_i (i = I, II, III, IV) above the vessel bottom. Five cylindrical regions of the agitated liquid volume were defined: region I of volume V_I occupied the space between the bottom ($z = 0$) and the horizontal cross-section at level z_I. Region II of volume V_{II} was located between z_I and z_{II} levels. Region III of volume V_{III} referred to the cylindrical space between z_{II} and z_{III} levels and contained the impeller. In region III it was additionally distinguished the impeller region V_m occupying the space of liquid flowing downwards. That impeller region had the form of a truncated cone. Regions IV (volume V_{IV}) and V (volume V_V) were located between following levels: z_{III} and z_{IV}, and z_{IV} and H, respectively.

In the present study the transport through the horizontal cross-sections of the agitated vessel at various z_i levels was considered. For the axisymmetrical velocity and pressure fields the following relations for the axial volumetric flow rate $\dot{V}_z$ and axial energy transport rate $\dot{E}_z$ should be valid.

$$\mathrm{d}\dot{V}_z = w_z \, 2\pi r \, \mathrm{d}r \,, \tag{1}$$

$$\mathrm{d}\dot{E}_z = \left(p_{st} + \frac{w^2 \varrho}{2} + \varrho g z \right) \mathrm{d}\dot{V}_z \,. \tag{2}$$

In the case of axial transfer the velocity w denotes the resultant mean velocity in the vertical plane of the given radial and axial coordinates. The sum of the static and dynamic pressures is the total pressure p_t indicated by the Pitot tube:

$$p_t = p_{st} + \frac{w^2 \varrho}{2} \,. \tag{3}$$

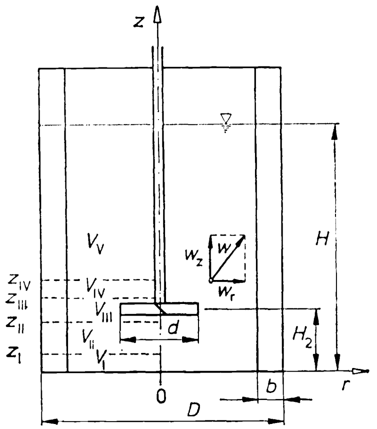

FIG. 1

Scheme of agitated system; $D = 0{\cdot}29$ m, $H = D$, $b = D/10$, $d/D = 1/3$, $1/4$ or $1/5$

The net axial flow rate (upward $\dot{V}_{zu}$ and downward $\dot{V}_{zd}$) through the given horizontal cross-section (z = const.) of the vessel must be equal to zero according to the continuity equation:

$$\Delta\dot{V}_z = \dot{V}_{zd} + \dot{V}_{zu} = 0\,, \tag{4}$$

$$\dot{V}_{zi} = \int_{r_{li}}^{r_{hi}} \mathrm{d}\dot{V}_z = 2\pi \int_{r_{li}}^{r_{hi}} r w_z \,\mathrm{d}r\,, \quad (i = u, d)\,. \tag{5}$$

The radii r_{li} and r_{hi} stand for the lower and higher radial coordinate of the zone where the upward ($i = u$) or downward ($i = d$) flow takes place at the chosen level z. In the space below impeller, where the reversal flow in a conical space under the impeller appeared, two zones of the upward flow exist.

Analogously, the upward and downward liquid streams convey the mechanical energy with the rates $\dot{E}_{zu}$ and $\dot{E}_{zd}$ given by Eq. (*6*) and resulted in the net axial energy flow $\Delta\dot{E}_z$:

$$\dot{E}_{zi} = \int_{r_{li}}^{r_{hi}} \mathrm{d}\dot{E}_z = \int_{r_{li}}^{r_{hi}} (p_t + \varrho g z)\,\mathrm{d}\dot{V}_z\,, \quad (i = u, d)\,, \tag{6}$$

$$\Delta\dot{E}_z = \dot{E}_{zu} + \dot{E}_{zd}\,. \tag{7}$$

It is convenient to separate the total pressure p_t and hydrostatic pressure $\varrho g z$ because only the former one depends on r and on impeller tip speed $\pi d n$:

$$\dot{E}^*_{iz} = \int_{r_{li}}^{r_{hi}} p_t\,\mathrm{d}\dot{V}_z\,, \quad (i = u, d)\,. \tag{8}$$

For the given level of constant z the net volumetric flow $\Delta\dot{V}_z$ is zero and we can write Eq. (*7*) in the similar form

$$\Delta\dot{E}_z = \dot{E}^*_{zu} + \dot{E}^*_{zd}\,. \tag{9}$$

The rate of the dissipated energy $\dot{E}_{12}$ in the space between levels z_1 and z_2 can be determined from the difference of $\Delta\dot{E}_z$ values corresponding to the two levels chosen:

$$\dot{E}_{12} = \left|\Delta\dot{E}_{z1} - \Delta\dot{E}_{z2}\right|. \tag{10}$$

According to the previous assumptions, $\Delta\dot{E}_z$ is equal to zero in the case of $z = 0$ and $z = H$. The $\dot{E}_{12}$ energy is dissipated in volume V_{12}:

$$V_{12} = (\pi/4)\, D^2 |z_1 - z_2|\,. \tag{11}$$

Mean value ε_{12} of the dissipated mechanical energy per unit volume can be derived from following equation

$$\varepsilon_{12} = \dot{E}_{12}/V_{12}\,. \tag{12}$$

Mean value $\bar{\varepsilon}$ for the whole vessel volume $V = \pi D^2 H/4$ was also calculated by means of the impeller power consumption

$$N = Po n^3 d^5 \varrho , \tag{13}$$

so we have

$$\bar{\varepsilon} = \frac{N}{V} = \frac{4 P_o n^3 d^5 \varrho}{\pi D^2 H} . \tag{14}$$

Some of the studied quantities become independent of the impeller speed after their transformation into the dimensionless form. It can be achieved by dividing the used quantities by some products of impeller speed n and diameter d or D, in a way similar to previous papers, e.g. refs[3,4]:

$$R = 2r/D , \tag{15}$$

$$W_i = w_i/\pi dn , \quad (i = r, z) , \tag{16}$$

$$P_i = 2p_i/(\pi dn)^2 \varrho , \quad (i = st, t) , \tag{17}$$

$$K_{Vi} = \dot{V}_{zi}/nd^3 , \quad (i = u, d) , \tag{18}$$

$$K_{Ei} = 2\dot{E}^{*}_{zi}/\pi^2 n^3 d^5 \varrho , \quad (i = u, d) . \tag{19}$$

Using the above equations one can derive relations (*20*) and (*21*) which were used in this study to compute the integral measures of the volumetric and energy flow rates:

$$K_{Vi} = \frac{\pi^2}{2} \left(\frac{D}{d}\right)^2 \int_{R_{li}}^{R_{hi}} R W_z \, dR , \quad (i = u, d) , \tag{20}$$

$$K_{Ei} = \frac{\pi^2}{2} \left(\frac{D}{d}\right)^2 \int_{R_{li}}^{R_{hi}} R W_z P_t \, dR , \quad (i = u, d) . \tag{21}$$

The integration limits R_{li} and R_{hi} denote the dimensionless radial coordinates of the boundaries of the upward $(i = u)$ or downward $(i = d)$ flow direction of the agitated liquid.

Finally, the following relation can be obtained for the ratio of the mean energy dissipation rates in the volume between z_1 and z_2 levels and in the whole agitated vessel:

$$\frac{\varepsilon_{12}}{\bar{\varepsilon}} = \frac{\pi^2 H}{2Po(z_1 - z_2)} \left[(K_{Ed})_{z_1} - (K_{Ed})_{z_2} - (K_{Eu})_{z_1} + (K_{Eu})_{z_2}\right] . \tag{22}$$

When dealing with the V_{III} volume the term $2Po/\pi^2$ should be added in the square brackets becaue of the impeller power input. The dimensionless form of the results presentation was chosen because the K_{Ei} values were practically independent of the impeller speed.

EXPERIMENTAL

The flow measurements were carried out in the baffled vessel of the inner diameter $D = 0{\cdot}29$ m, see Fig. 1. The pitched blade turbine impeller of the varied diameter $d = D/3$; $D/4$; $D/5$ was used. The impeller had 6 blades inclined under 45° and pumped the agitated liquid toward the vessel bottom. The impeller centre was located at the level $H_2 = D/4$ above the vessel bottom. Two or three impeller speeds were applied for each of the three impellers. The flow of the agitated liquid (distilled water) was always turbulent, i.e. $Re > 10^4$. The measurement technique of the three-holes Pitot tube was employed. Its theory and methodology has been described elsewhere[3,5].

The measurements performed in four cross-sections at the levels $z_I = 0{\cdot}02$ m, $z_{II} = 0{\cdot}06$ m, $z_{III} = 0{\cdot}09$ m, $z_{IV} = 0{\cdot}11$ m were considered in the presented paper. All the measurement points were located in the vertical plane dividing symmetrically the distance between two adjacent baffles. Some fragments of the experimental results have been reported previously[1,3,6]. The measurements were carried out for each z level in at least 15 points of different radial coordinate. The data were used to derive radial profiles of the mean velocity components; radial w_r and axial w_z, and static p_{st} and total p_t pressures at the chosen level.

RESULTS AND DISCUSSION

The preceding papers in this Series did not contain information on the radial distribution of the mean velocity components and pressures for the level $z_{IV} = 0{\cdot}11$ m. The distributions of the dimensionless radial and axial velocity components, the static and total pressures were presented graphically in Figs 2–5 for that level and three impeller sizes used. All the presented profiles were qualitatively similar to those for z_{III} except for the total pressure profile close to the wall. The well established rule of the independence of local values of the normalized components W_i (see Eq. (*16*)) of the mean velocity on the impeller speed n was confirmed. Their profiles, however, were markedly influenced by the impeller diameter d resulting in a decreased W_i magnitude for smaller d. The same remarks were valid for the normalized static pressure P_{st} and total pressure P_t. The scatter of the experimental results for various n can be attributed to the measurement errors caused by the turbulent flow, as discussed in the papers[3,4].

In order to show the local contribution to the total mass flux and its mechanical energy the profiles of integrands RW_z and RW_zP_t appearing in the integrals (*20*) and (*21*) were constructed and presented in Figs 6, 7. To facilitate their comparison the profiles were collected for all d/D ratios and z_i levels aplied.

The dimensionless local W_z values generally diminished their magnitude when the d/D ratio was decreased. A fair consistency of the absolute values of K_{Vi} for the upward and downward flow was obtained, cf. Table I.

Maximum axial flow rate which corresponds to the total circulation flow rate was found for z_{II} and z_{III}. The flow rate data were approximately 25% higher than those reported in the literature[7]. The maximum error of 25% of the K_{Vi} value was estimated in this study.

Some qualitative differences can be observed between the W_z and RW_z profiles for the liquid stream leaving rotating impeller. The profile of absolute W_z values exhibited a sharp maximum[8] at $r/d \approx 0{\cdot}4$ but a maximum plateau of $|RW_z|$ profile was obtained for approximate range $r/d \in [0{\cdot}4; 0{\cdot}5]$. That conclusion is valid for all the impeller diameters applied. Analyzing the RW_z profiles for z_{II} and z_{III} one can also recognize a strong suction effect of the impeller because of shifting the $|RW_z|$ maximum in the downward flow region toward the vessel axis. The points of the flow direction reversal from the upward flow (wall jet) to the downward flow turn up in the R range from 0·65 to 0·75. Also a conical zone of the induced reversal flow under the impeller was found but its existence was negligible for the total flow balances.

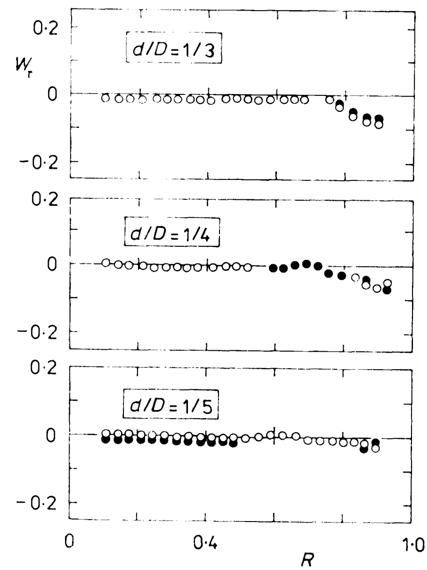

FIG. 2

Radial profiles of the radial component of mean velocity. $d/D = 1/3$: ○ $n = 5{,}8\ s^{-1}$, ● $n = 8{,}3\ s^{-1}$; $d/D = 1/4$: ○ $n = 10\ s^{-1}$, ● $n = 15\ s^{-1}$; $d/D = 1/5$: ○ $n = 22\ s^{-1}$, ● $n = 28\ s^{-1}$

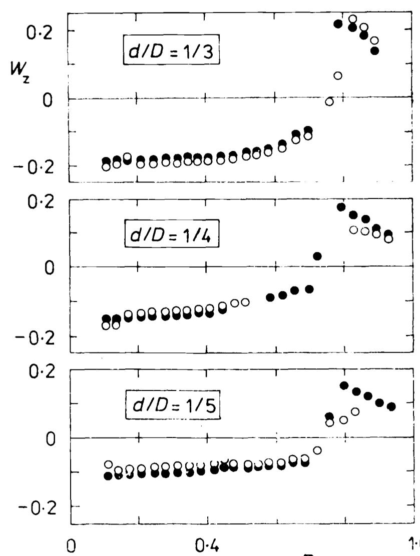

FIG. 3

Radial profiles of the axial component of mean velocity; legend as for Fig. 2

The impeller discharge stream conveyed the highest energy amount, cf. the 5-fold scale magnification for z_{II} in Fig. 7. Maximum energy was conveyed axially by the liquid with the highest velocity because the radial coordinates of the peaks of the RW_zP_t and W_z profiles were practically equal. But more than two local extremum points of the profile of the energy flow rate were often found because the total

TABLE I

Axial volumetric flow characteristics K_{vi}

Level z	K_{vi}					
	$d/D = 1/3$		$d/D = 1/4$		$d/D = 1/5$	
	$i = d$	$i = u$	$i = d$	$i = u$	$i = d$	$i = u$
z_I	−1·13	1·00	−1·57	1·63	−2·12	2·36
z_{II}	−2·05	1·87	−2·59	2·84	−3·36	3·03
z_{III}	−1·99	1·91	−2·69	2·69	−3·42	3·41
z_{IV}	−1·65	1·53	−2·15	2·22	−2·49	2·44

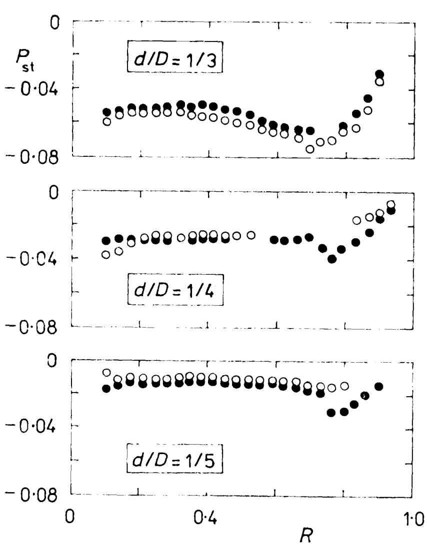

FIG. 4

Radial profiles of the static pressure; legend as for Fig. 2

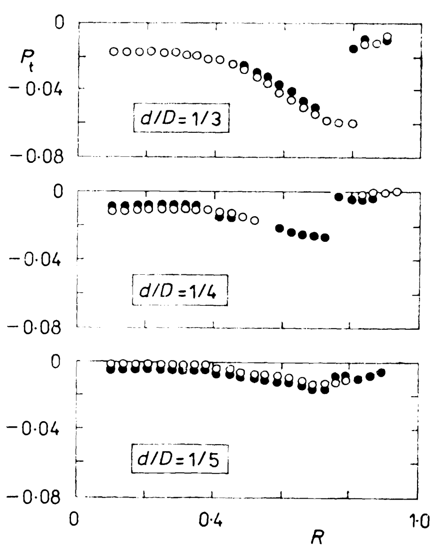

FIG. 5

Radial profiles of the total pressure; legend as for Fig. 2

pressure P_t was not always negative. An analysis of the experimental data allowed one to state that a region of the positive total pressure prevailed in the discharged impeller stream, at the main part of the vessel bottom and in the wall jet up to z_{III}. That region is characterized by high liquid velocities and velocity gradients. The P_t values are negative in the remainder of the agitated charge, i.e. above the impeller, in the induced flow region and in the reversal flow region under the impeller.

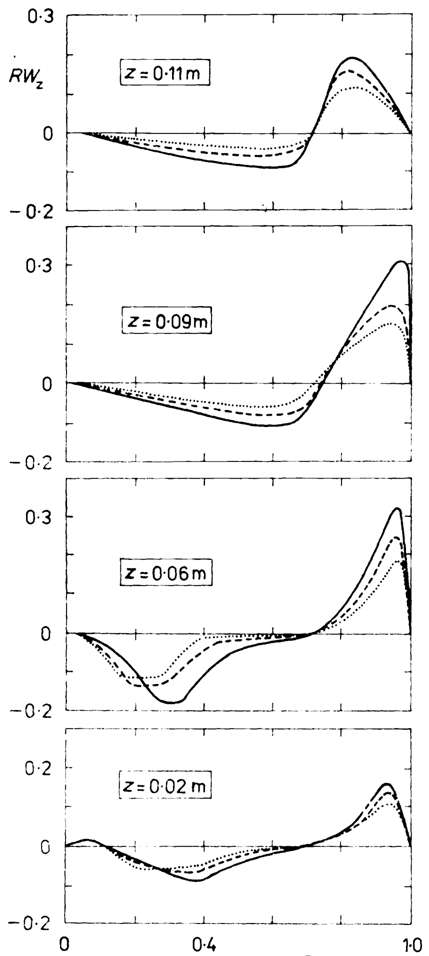

FIG. 6

Radial profiles of the RW_z expression; ——— $d/D = 1/3$, – – – $d/D = 1/4$, ······ $d/D = 1/5$

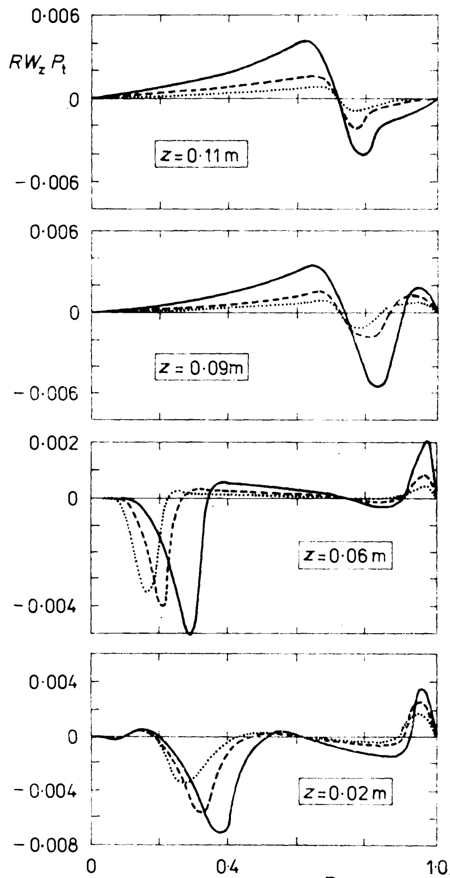

FIG. 7

Radial profiles of the RW_zP_t expression; ——— $d/D = 1/3$, – – – $d/D = 1/4$, ······ $d/D = 1/5$

The computed integral characteristics K_{Ei} were collected in Table II for various d/D ratios and regions in the vessel. One can conclude that the energy dissipation rate in the liquid above the impeller was smaller than the rate for other regions by

TABLE II
Axial energy flow characteristics

Levels z_1	Levels z_2	Volume	d/D	$(K_{Ei})_{z_1} - (K_{Ei})_{z_2}$, $i = d$	$(K_{Ei})_{z_1} - (K_{Ei})_{z_2}$, $i = u$	$\varepsilon_{12}/\bar{\varepsilon}$	$\bar{\varepsilon}_{12}/\bar{\varepsilon}$
			1/3	−0·010	−0·046	1·5	
z_I	0	V_I	1/4	0·004	−0·042	1·9	1·8
			1/5	0·012	−0·035	1·7	
			1/3	0·022	−0·157	2·5	
z_{II}	0	$V_I + V_{II}$	1/4	0·015	−0·179	2·7	2·7
			1/5	0·015	−0·191	2·9	
			1/3	0·306[a]	0·195	3·1	
z_{III}	z_{II}	V_{III}	1/4	0·325[a]	0·202	3·4	3·2
			1/5	0·327[a]	0·217	3·1	
			1/3	−0·016	0·038	0·2	
z_{III}	H	$V_{IV} + V_V$	1/4	−0·004	0·023	0·1	
			1/5	−0·002	0·026	0·1	
							0·2
			1/3	−0·024	0·042	0·3	
z_{IV}	H	V_V	1/4	−0·009	0·031	0·2	
			1/5	−0·011	0·027	0·2	

[a] The value of $2 \cdot Po/\pi^2$ was added because of power input to the impeller region.

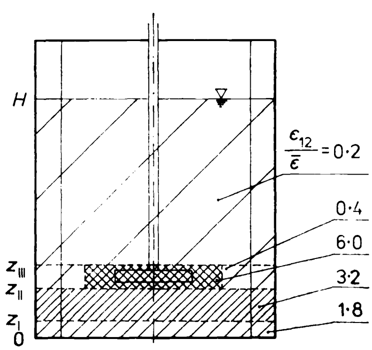

FIG. 8
Sketch of the energy dissipation rate distribution in an agitated vessel

one order of magnitude. The evaluation precision of a K_{Ei} value was limited owing to the experimental technique used and the maximum error of $K_{Ei} = 0{\cdot}02$ was accepted which roughly corresponds to $\varepsilon_{12}/\bar{\varepsilon} = 0{\cdot}1$. This was a reason that an averaged value of $\varepsilon_{12}/\bar{\varepsilon} = 0{\cdot}2$ was proposed for regions of V_{IV} and V_V volume above the impeller. The power number $Po = 1{\cdot}7$ was adopted[9].

It was not possible to compute the energy dissipation rate separately for the downward and upward flow streams because of lack of the data for the energy transfer between those streams. An exception was made for the impeller region – the downward flow part of the region III to estimate the hydraulic efficiency of the impellers tested. The efficiency was defined[1] as the ratio of the energy transferred to the liquid by the impeller related to the energy N supplied to the impeller. The doubled value of the $\varepsilon_{12}/\bar{\varepsilon}$ ratio for the regions above the impeller, i.e. 0·4, was assumed to be valid for the upward flow part of the region III. Thus the hydraulic impeller efficiency amounted to 70%; 66%; 70% for $d/D = 1/3$; $1/4$; $1/5$, respectively.

That assumption enables one to estimate the mean energy dissipation rate $\varepsilon_{12}/\bar{\varepsilon} = 6{\cdot}0$ in the impeller region. The mean rate $\varepsilon_{12}/\bar{\varepsilon} = 3{\cdot}2$ for the V_{II} volume (levels z_{II} to z_{III}) was obtained on the basis of the data for the V_I and V_{II} volumes (cf. Table II). The obtained energy dissipation rates are presented graphically in Fig. 8. The fraction of the impeller energy which is dissipated in the main parts of the vessel was also estimated. The 32% contribution to the total energy dissipation was attributed to the impeller region (V_m volume), 54% to the space below the impeller (V_I and V_{II} volumes) and 14% to the reminder of the agitated charge. These numbers were very similar to those by Ranade et al.[2] and supported the reliability of the data presented. All the data on the energy dissipation can essentially contribute to the mathematical modelling of turbulent transport processes in agitated vessels with pitched blade turbine impellers. They may be used to determine the distribution of eddy viscosity or diffusivity in agitated liquid. Knowledge of the local energy dissipation rate is also of great importance for mass transfer in two or three phase agitated systems.

SYMBOLS

D	vessel diameter, m
d	impeller diameter, m
$\dot{E}_z$	axial energy flow rate, Eq. (*7*), W
$\dot{E}_z^*$	axial energy flow rate, Eq. (*8*), W
H	liquid height in the vessel, m
H_2	height of impeller center above vessel bottom, m
K_E	axial energy flow number, Eq. (*19*)
K_V	axial volumetric flow number, Eq. (*18*)
N	impeller power consumption, W
n	impeller rotational speed, s^{-1}
P	non-dimensional pressure, Eq. (*17*)

p	agitated liquid pressure, N m^{-2}
R	non-dimensional radial coordinate, Eq. (*15*)
r	radial coordinate, m
V	volume, m^3
$\dot{V}_z$	axial volumetric flow rate, Eq. (*5*), m^3 s^{-1}
W	non-dimensional mean velocity component, Eq. (*16*)
w	velocity, m s^{-1}
z	axial coordinate, m
Po	impeller power number, $Po = N/n^3d^5\varrho$
ε	energy dissipation rate, W m^{-3}
ϱ	liquid density, kg m^{-3}

Subscripts and superscripts

d	downward flow
r	radial component
st	static
t	total
u	upward flow
z	axial component
1, 2	levels investigated
I – V	cross-section or region of agitated vessel
$\bar{}$	averaged quantity

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